EVOLUTIONARY DYNAMICS OF CLIMATE CHANGE UNDER COLLECTIVE-RISK DILEMMAS

FRANCISCO C. SANTOS
Departamento de Informática & Centria, Universidade Nova de Lisboa,
2829-516 Caparica, Portugal.
and
ATP-group, Centro de Matemática e Aplicações Fundamentais,
Instituto para a Investigação Interdisciplinar, P-1649-003 Lisboa Codex, Portugal

VÍTOR VASCONCELOS, MARTA D. SANTOS *and* PEDRO NEVES
ATP-group, Centro de Matemática e Aplicações Fundamentais,
Instituto para a Investigação Interdisciplinar, P-1649-003 Lisboa Codex, Portugal

JORGE M. PACHECO[†]
Departamento de Matemática e Aplicações, Universidade do Minho,
4710 - 057 Braga, Portugal.
and
ATP-group, Centro de Matemática e Aplicações Fundamentais,
Instituto para a Investigação Interdisciplinar, P-1649-003 Lisboa Codex, Portugal

Preventing global warming is a public good requiring overall cooperation. Contributions will depend on the risk of future losses, which plays a key role in decision-making. Here, we discuss an evolutionary game theoretical model in which decisions within small groups under high risk and stringent requirements toward success significantly raise the chances of coordinating to save the planet's climate, thus escaping the tragedy of the commons. We discuss both deterministic dynamics in infinite populations, and stochastic dynamics in finite populations.

1. Introduction

In a dance that repeats itself cyclically, countries and citizens raise significant expectations every time a new International Environmental Summit is settled. Unfortunately, few solutions have come out of these colossal and flashy meetings, challenging our current understanding and models on decision-making, so that more effective levels of discussion, agreements and coordination

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become accessible. From Montreal and Kyoto to Copenhagen summits, it is by now clear how difficult it is to coordinate efforts [1, 2]. Often, individuals, regions or nations opt to be *free riders*, hoping to benefit from the efforts of others while choosing not to make any effort themselves. Cooperation problems faced by humans often share this setting, in which the immediate advantage of free riding drives the population into the tragedy of the commons [3], the ultimate limit of widespread defection [3-12].

To address this and other cooperation conundrums, ubiquitous at all scales and levels of complexity, the last decades have witnessed the discovery of several core mechanisms responsible to promote and maintain cooperation at different levels of organization [3, 5, 10, 13-26]. Most of these key principles have been studied within the framework of two-person dilemmas such as the Prisoner's Dilemma, which constitutes a powerful metaphor to describe conflicting situations often encountered in the natural and social sciences. Many real-life situations, however, are associated with collective action based on joint decisions made by a group often involving more than two individuals [3, 5, 13, 27]. These types of problems are best dealt-with in the framework of N-person dilemmas and Public Goods games, involving a much larger complexity that only recently started to be unveiled [5, 14, 22, 28-33]. The welfare of our planet accounts for possibly the most important and paradigmatic example of a public good: a global good from which everyone profits, whether or not they contribute to maintain it.

One of the most distinctive features of this complex problem, only recently tested and confirmed by means of actual experiments [9], is the role played by the perception of risk that accrues to all actors involved when making a decision. Indeed, experiments confirm the intuition that the risk of collective failure plays central role in dealing with climate change. Up to now, the role of risk has remained elusive [1, 2, 11]. Additionally, it is also unclear what is the ideal scale or size of the population engaging in climate summits — whether game participants are world citizens, regions or country leaders —, such that the chances of cooperation are maximized. Here we address these two issues in the context of game theory and population dynamics.

The conventional public goods game – the so-called N-person Prisoner's Dilemma – involve a group of N individuals, who can be either Cooperators (C) or Defectors (D). Cs contribute a cost "c" to the public good, whereas Ds refuse to do so. The accumulated contribution is multiplied by an enhancement factor that returns equally shared among all individuals of the group. This implies a collective return which increases linearly with the number of contributors, a situation that contrasts with many real situations in which performing a given task requires the cooperation of a minimum number of individuals of that group [28-30, 33-38]. This is the case in international environmental agreements which demand a minimum number of ratifications to come into practice [1, 2, 9, 39-42], but examples abound where a minimum number of individuals, which does not necessarily equal the entire group, must simultaneously cooperate before any outcome (or public good) is produced [28, 29]. Furthermore, it is by now clear that the N-person Prisoner's Dilemma fails short to encompass the role of risk, as much as the non-linearity of most collective action problems.

Here we address these problems resorting to a simple mathematical model, adopting unusual concepts within political and sustainability science research, such as peer-influence and evolutionary game theory [14, 43, 44]. As a result we encompass several of the key elements stated before regarding the climate change conundrum in a single dynamical model.

In the following we show how small groups under high risk and stringent requirements toward collective success significantly raise the chances of coordinating to save the planet's climate, thus escaping the tragedy of the commons. In other words, global cooperation is dependent on how aware individuals are concerning the risks of collective failure and on the pre-defined premises needed to accomplish a climate agreement. Moreover, we will show that to achieve stable levels of cooperation, an initial critical mass of cooperators is needed, which will then be seen as role models and foster cooperation.

We will start by presenting the model in Section 2. In Section 3 we discuss the situation in which evolution is deterministic and proceeds in very large populations. In Section 4 we analyze the evolutionary dynamics of the same dilemma in finite populations under errors and behavioral mutations. Finally, in Section 5 we provide a summary and concluding remarks.

2. Model

Let us consider a large population of size Z, in which individuals engage in a N-person dilemma, where each individual is able to contribute or not to a common good, i.e., to cooperate or to defect, respectively. Game participants have each an initial endowment b. Cooperators (Cs) contribute a fraction c of their endowment, while defectors (Ds) do not contribute. As previously stated, irrespectively of the scale at which agreements are tried, most demand a minimum number of contributors to come into practice. Hence, whenever parties fail to achieve a previously defined minimum of contributions, they may fail to achieve the goals of such agreement (which can also be understood as the benefit "*b*"), being this outcome, in the worst possible case, associated with an appalling doomsday scenario. To encompass this feature in the model we require a minimum collective investment to ensure success: If the group of size *N* does not contain at least *M* *C*s (or, equivalently, a collective effort of *Mcb*), all members will lose their remaining endowments with a probability *r* (the *risk*); otherwise everyone will keep whatever they have. Hence, $M<N$ represents a coordination threshold [9, 28], necessary to achieve a collective benefit. As a result, the average payoff of a *D* in a group of size *N* and *k* *C*s can be written as

$$\Pi_D(k) = b\left\{\theta(k-M) + (1-r)\left[1-\theta(k-M)\right]\right\}, \tag{1}$$

where $\theta(x)$ is the Heaviside step function ($\theta(x<0)=0$ and $\theta(x\geq 0)=1$). Similarly, the average payoff of a ***C*** is given by

$$\Pi_C(k) = \Pi_D(k) - cb\,. \tag{2}$$

The risk *r* is here introduced as a probability, such that with probability *(1-r)* the benefit will be collected independent of the number of contributors in a group.

This collective-risk dilemma represents a simplified version of the game used in the experiments performed by Milinski et al [9] on the issue of the mitigation of the effects of climate change, a framework which is by no means the standard approach to deal with International Environmental Agreements and other problems of the same kind [1, 2, 39, 40]. The present formalism has the virtue of depicting black on white the importance of risk and its assessment in dealing with climate change, something that Heal et al [41, 45] have been conjecturing for quite a while. At the same time, contrary to the experiments in [9], our analysis is general and not restricted to a given group size.

Additionally, and unlike most treatments [1], our analysis will not rely on individual or collective rationality. Instead, our model relies on evolutionary game theory combined with one-shot public goods games, in which errors are allowed. In fact, our model includes what we believe are key factors in any real setting, such as bounded rational individual behavior, peer-influence and the importance of risk assessment in meeting the goals defined from the outset.

We assume that individuals tend to copy others whenever these appear to be more successful. Contrary to strategies defined by a contingency plan which, as argued before [46], are unlikely to be maintained for a long time scale, this social learning (or evolutionary) approach allows policies to change as time goes by [22, 47, 48], and likely these policies will be influenced by the behavior (and achievements) of others, as previously shown in the context of donations to public goods [44, 49, 50]. This also takes into account the fact that agreements may be vulnerable to renegotiation, as individuals may agree on intermediate goals or assess actual and future consequences of their choices to revise their position [1, 2, 7, 39, 40, 45].

3. Evolution of collective action in large populations

In the framework of evolutionary game theory, the evolution or social learning dynamics of the fraction x of Cs (and $1-x$ of Ds) in a large population ($Z \to \infty$) is governed by the gradient of selection associated with the replicator dynamics equation [14, 28, 51]

$$g(x) \equiv \dot{x} = x(1-x)\left(f_C(x) - f_D(x)\right) \ , \tag{3}$$

which characterizes the behavioral dynamics of the population, where f_C (f_D) is the fitness of Cs (Ds), here associated with the game payoffs. According to the replicator equation, Cs (Ds) will increase in the population whenever $g(x)>0$ ($g(x)<0$). If one assumes an unstructured population, where every individual can potentially interact with everyone else, the fitness (or social success) of each individual can be obtained from a random sampling of groups. The latter leads to groups whose composition follows a binomial distribution. Hence, we may write the fitness of Cs, f_C, and Ds, f_D, as [28-30]

$$f_C(x) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \Pi_C(k+1) \tag{4a}$$

and

$$f_D(x) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \Pi_D(k) \ , \tag{4b}$$

where $\Pi_C(k)$($\Pi_D(k)$) stands for the payoff of a C (D) in a group of size N and k Cs, as defined above in equations (1) and (2).

Fig. 1 shows that, in the absence of risk, $g(x)$ is always negative. Risk, in turn, leads to the emergence of two mixed internal *equilibria*, rendering cooperation viable: for finite risk r, both Cs (for $x<x_L$) and Ds (for $x>x_R$) become disadvantageous when rare. Co-existence between Cs and Ds becomes stable at a fraction x_R which increases with r. Collective coordination becomes easier to achieve under high-risk and, once the coordination barrier (x_L) is overcome, high levels of cooperation will be reached.

The appearance of two internal *equilibria* under risk can be studied analytically, as the roots of the fitness difference $Q(x) \equiv f_C(x) - f_D(x)$

determines the occurrence of non-trivial *equilibria* of the replicator dynamics. From the equations above we may write, after some algebra, that

$$Q(x) = b\left[\binom{N-1}{M-1} x^{M-1}(1-x)^{N-M}\, r - c\right]. \tag{5}$$

Defining the cost-to-risk ratio $\gamma = c/r$, i.e., the ratio between the fraction of the initial budget invested by every C and the risk of losing it, the sign of *Q(x)* is conveniently analyzed by using the polynomial

$$p(x) = \binom{N-1}{M-1} x^{M-1}(1-x)^{N-M} - \gamma, \tag{6}$$

which, in turn, can be used to determine the critical value $\bar{\gamma}$ below which an interior fixed point $x^* \in (0,1)$ emerges. Indeed, we can prove the following theorem.

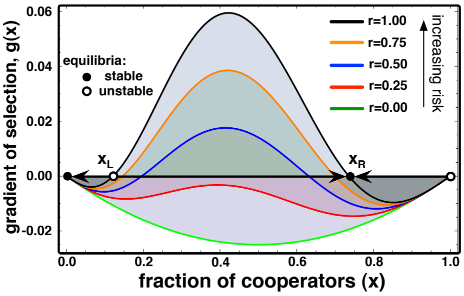

Fig. 1. For each fraction of *C*s, if the gradient *g(x)* is positive (negative) the fraction of *C*s will increase (decrease). Increasing risk (*r*) modifies the population dynamics rendering cooperation viable depending on the initial fraction of *C*s (N=6, *M=3* and *c=0.1*).

Theorem 1. Let $\Gamma(x) = \binom{N-1}{M-1} x^{M-1}(1-x)^{N-M}$. For *1<M<N*, there exists a critical cost-to-risk ratio $\bar{\gamma} = \Gamma(\bar{x}) > 0$ and fraction of *C*s $0 < \bar{x} < 1$ such that

1. If $\gamma > \bar{\gamma}$, the evolutionary dynamics has no interior *equilibria*.
2. If $\gamma = \bar{\gamma}$, then $\bar{x}$ is a unique interior equilibrium, being this equilibrium unstable.
3. If $\gamma < \bar{\gamma}$, there are two interior equilibria $\{x_L, x_R\}$, such that $x_L < \bar{x} < x_R$, x_L is unstable and x_R stable.

Proof. Let us start by noticing that $\dfrac{d\,\Gamma(x)}{dx} = -\begin{pmatrix} N-1 \\ M-1 \end{pmatrix} x^{M-2}(1-x)^{N-M-1} s(x)$, where $s(x) = 1 + (N-1)x - M$. Since $N > 2$ and $1 < M < N$, then $d\,\Gamma(x)/dx$ has a single internal root for $\bar{x} = (M-1)/(N-1)$. In addition, *s(x)* is negative (positive) for $x < \bar{x}$ ($x > \bar{x}$), which means that Γ has a global maximum for $x = \bar{x}$.

1) and *2)* can now easily follow. Since Γ has a maximum at $\bar{x}$, it follows that $\Gamma(x) = 0$ has no solutions for $\gamma > \bar{\gamma}$ and a single one, at $\bar{x}$, for $\gamma = \bar{\gamma}$. Moreover, both when $x \to 0$ and $x \to 1$, $p(x) < 0$, making $x = 0$ a stable fixed point and $x = 1$ an unstable one. Therefore, if $\bar{x}$ is a root, it must be unstable.

To prove *c)*, we start by noticing that $\Gamma(0) = \Gamma(1) = 0$. From the sign of *s(x)* (see above), $\Gamma(x)$ is clearly monotonic increasing (decreasing) to the left (right) of $\bar{x}$. Hence, there is a single root x_L (x_R) in the interval $0 < x < \bar{x}$ ($\bar{x} < x < 1$). Since $x = 0$ is stable and $x = 1$ unstable, x_R must be stable and x_L unstable. ☐

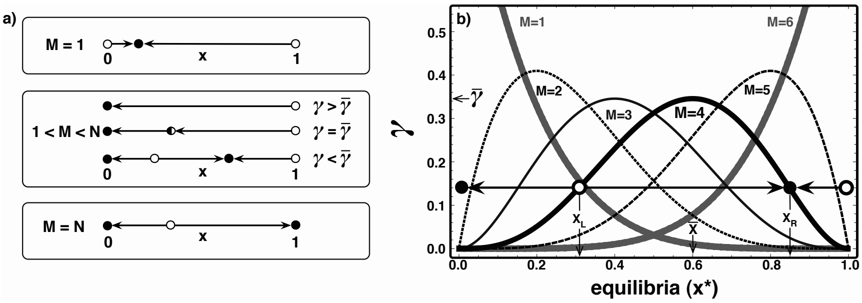

Figure 2. a) Classification of all possible dynamical scenarios when evolving an infinitely large population of *C*s and *D*s as a function of *γ*, *M* and N. A fraction *x* of an infinitely large population adopts the strategy *C*; the remaining fraction *1-x* adopts *D*. The replicator equation describes the evolution of *x* over time. Solid (open) circles represent stable (unstable) *equilibria* of the evolutionary dynamics; arrows indicate the direction of selection. b) Internal roots *x** of *g(x)* for different values of the cost-to-risk ratio *γ=c/r*, at fixed group size (N=6) and different coordination thresholds (*M*). For each value of *γ* one draws a horizontal line; the intersection of this line with each curve gives the value(s) of *x**, defining the internal *equilibria* of the replicator dynamics. The empty circle represents an unstable fixed point (x_L) and the full circle a stable fixed point (x_R) (*M=4* and *γ=0.15* in example).

Theorem 2. For $M = 1$, if $\gamma < \bar{\gamma}$, there is one stable interior equilibrium point in the interval $0 < x < 1$.

Proof. If $M = 1$, $\Gamma(x) = (1-x)^{N-1}$, which is a monotonic decreasing function for $0 < x < 1$. This means that the function $p(x)$ has only one zero in that interval, i.e., there is only one $\bar{x}\,(0<\bar{x}<1)$ such that $p(\bar{x})=0$. Given that $p(x)$ is positive (negative) for $x<\bar{x}$ ($x>\bar{x}$) then $\bar{x}$ is a stable equilibrium point.

Theorem 3. For $M=N$, if $\gamma<\bar{\gamma}$, there is one unstable interior equilibrium point in the interval $0<x<1$.

Proof. If $M=N$, $\Gamma(x)=x^{N-1}$, which is a monotonic increasing function for $0<x<1$. This means that the function $p(x)$ has only one zero in that interval, i.e., there is only one $\bar{x}\,(0<\bar{x}<1)$ such that $p(\bar{x})=0$. Given that $p(x)$ is negative (positive) for $x<\bar{x}$ ($x>\bar{x}$) then $\bar{x}$ is an unstable equilibrium point.

In Fig. 2a, we provide a concise scheme of all possible dynamical scenarios that emerge from collective-risk dilemmas, showing how the coordination threshold and the level of risk play a central role in dictating the viability of cooperation. Fig. 2b also shows the role played by the threshold M: for fixed (and low) γ, increasing M will maximize cooperation (increase of x_R) at the expense of making it more difficult to emerge (increase of x_L).

4. Evolution of collective action in small populations

Real populations are finite and often rather small, contrary to the hypothesis underlying the dynamics portrayed in Section 3. In particular, this is the case of the famous world summits where group and population sizes are comparable and of the order hundreds, as individuals are here associated with nations or their respective leaders. For such population sizes, stochastic effects play an important role and the deterministic description of the previous section may be too simplistic [52].

For finite, well-mixed populations of size *Z*, the binomial sampling in equations (4) is replaced by a hyper-geometric sampling (sampling without replacement). As a result, the average fitness of *D*s and *C*s in a population with *k* *C*s, is now written as

$$f_D(k)=\binom{Z-1}{N-1}^{-1}\sum_{j=0}^{N-1}\binom{k}{j}\binom{Z-k-1}{N-j-1}\Pi_D(j) \tag{7}$$

and

$$f_C(k)=\binom{Z-1}{N-1}^{-1}\sum_{j=0}^{N-1}\binom{k-1}{j}\binom{Z-k}{N-j-1}\Pi_C(j+1)\ . \tag{8}$$

respectively. We adopt a stochastic birth-death process [53] combined with the pair-wise comparison rule [54] in order to describe the social dynamics of *C*s (and *D*s) in a finite population. Under pair-wise comparison, each individual *i* adopts the strategy of a randomly selected member of the population *j* with probability given by the Fermi function (from statistical physics)

$$p_{ij} = \frac{1}{1+e^{-\beta(f_j - f_i)}} \ . \tag{9}$$

Here β controls the intensity of selection. For $\beta << 1$, selection is weak and individual fitness is but a small perturbation to random drift in behavioral space. Under this regime one recovers the replicator equation in the limit $Z \to \infty$ [54]. For arbitrary β, the quantity *g(x)* of Eq. (3), specifying the gradient of selection, is replaced in finite populations by [54]

$$G(k) \equiv T^+(k) - T^-(k) = \frac{k}{Z}\frac{Z-k}{Z}\tanh\left\{\frac{\beta}{2}\left[f_C(k) - f_D(k)\right]\right\}, \tag{10}$$

where *k* stands for the total number of *C*s in the population and

$$T^{\pm}(k) = \frac{k}{Z}\frac{Z-k}{Z}\left[1+e^{\mp\beta\left[f_C(k)-f_D(k)\right]}\right]^{-1} \tag{11}$$

for the probabilities to increase and decrease the number of *C*s in the population.

4.1. *Fixation probabilities*

The fact that, in finite populations, the continuous gradient of selection *g(x)* is replaced by a discrete *G(k/Z)* has implications in the overall evolutionary dynamics of the population. Importantly, in the absence of mutations evolutionary dynamics in finite populations will only stop whenever the population reaches a monomorphic state [52, 54]. Hence, in addition to the analysis of the shape of *G(k/Z)*, often one of the quantities of interest in studying the evolutionary dynamics in finite populations is the probability ϕ_k that the system fixates in a monomorphic cooperative state, starting from, for instance, a given number *k* of Cs. The fixation probability of *k* *C*s (ϕ_k) depends on the ratio $\lambda_j = T^-(j)/T^+(j)$, being given by [53]

$$\phi_k = \sum_{i=0}^{k-1}\prod_{j=1}^{i}\lambda_j \bigg/ \sum_{i=0}^{Z-1}\prod_{j=1}^{i}\lambda_j \ . \tag{12}$$

Under neutral selection (that is, in the limit $\beta \to 0$) the fixation probability trivially reads $\phi_k^N = k/Z$, providing a convenient reference point [17, 53-55]. For a given *k*, whenever $\phi_k > \phi_k^N$, natural selection will favor cooperative behavior, the opposite being true when $\phi_k < \phi_k^N$.

In Fig. 3 we plot the fixation probability as a function of the initial fraction of *C*s for different values of risk, and a population of 50 individuals. Even if cooperators remain disadvantageous for a wide range of the discrete frequency of *C*s (see Fig. 1), the fixation probability of *k C*s outperforms ϕ_k (picture as a dashed grey line) for most values of *k/Z*. This is due to the stochastic nature of the imitation process, which allows the fixation of rare cooperators, even when they are initially disadvantageous. Hence, even without random exploration of strategies [56], simple errors in the imitation process (finite *β*) are enough to overcome the unstable fixed point shown in Fig. 2 and reach a more cooperative basin of attraction on the right-hand side of the gradient (see below). As a result, for high values of risk and large, but finite, populations, cooperation is by far the strategy most favored by evolution irrespectively of the initial fraction of cooperators.

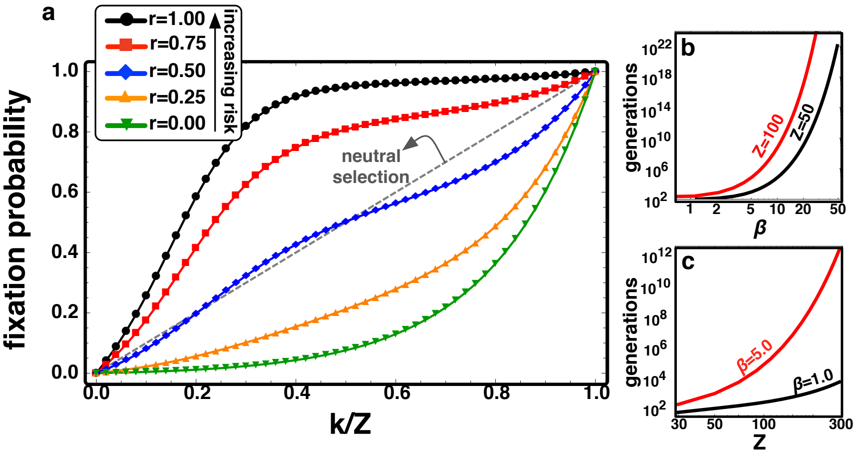

Figure 3. Evolutionary dynamics for different values of risk in finite populations. In panel a), we show the fixation probabilities for different values of risk (*r*) as a function of the number of Cs (*Z=50*, *c=0.1*, *N=6=2M*, *β=1.0*). In panels b) and c), we show the average number of generations (t_f/Z) [57, 58] needed to fixate an initial fraction of 0.5 of cooperators, as a function of the intensity of selection *β* (panel *b*) and population size Z (panel c). We consider the case of maximum risk (*r*=1) for both *b)* and *c)* panels and *c=0.1*, *N=6=2M*. Even if high risk can turn the fixation of cooperators almost certain (as shown in panel a)), the time the population takes to reach such state can be arbitrarily long.

As discussed above, in finite populations the evolutionary dynamics becomes stochastic. Yet, even if fixation in one of the two absorbing states is certain (*k=0* and *k=Z*), the time required to reach it can be arbitrarily long. This is particularly relevant in the presence of basins of attraction with polymorphic stable configurations, which correspond to finite population analogues of co-existence equilibria in infinite populations. For high intensities of selection and/or large populations, the time required for fixation (t_j) can increase significantly. Following Antal and Scheuring [57], the average number of updates t_j the population takes to reach full cooperation, starting from *j* cooperators, can be written as [57, 59]

$$t_j = -t_1 \frac{\phi_1}{\phi_j} \sum_{k=j}^{N-1} \prod_{m=1}^{k} \lambda_m + \sum_{k=j}^{N-1} \sum_{l=1}^{k} \frac{\phi_l}{T^+(j)} \prod_{m=l+1}^{k} \lambda_m \ , \qquad (13a)$$

where

$$t_1 = \sum_{k=j}^{N-1} \prod_{l=1}^{k} \frac{\phi_l}{T^+(l)} \prod_{m=l+1}^{k} \lambda_m \ . \qquad (13b)$$

This is illustrated in Fig.3b and Fig.3c, where we compute average number of generations (t_j / Z) needed to attain monomorphic cooperative state as a function of the intensity of selection and population size, starting from 50% of *C*s and *D*s for a dilemma with highest risk (*r=1*). These panels clearly indicate that even if high risk can turn the fixation of cooperators almost certain (as shown in the left panel), the time the population takes to reach such state can be arbitrarily long. In other words, while the computation of the fixation probabilities can be mathematically attractive, its relevance may be limited for large intensities of selection and/or large *Z*. In other words, the stochastic information built in ϕ_k shows how unstable roots of *G* may be irrelevant; however, the lack of time information in ϕ_k ignores the key role played by the stable roots of *G*.

Moreover, stochastic effects in finite populations can be of different nature, going beyond errors in the imitation process. One can also consider mutations, random exploration of strategies or any other reason that leads individuals to change their behavior, in addition to social learning by imitation dynamics [56]. In the simplest scenario, this creates a modified set of transition probabilities, with an additional random factor encoding the probability of a mutation (μ) in each update step. Under these circumstances, the population will never fixate in none of the two possible *monomorphic* states.

4.2. *Stationary distributions*

As discussed in the previous section, the existence of a stable equilibrium may turn the analysis of the fixation probability misleading. Not only fixation probabilities fail to characterize in a reasonable way the evolutionary dynamics under general conditions, if one considers other forms of stochastic effects as random exploration of strategies, the system will never fixate.

A proper alternative which overcomes the drawbacks identified in both ϕ_k and G consists in the analysis of the stationary distributions of the complete Markov chain *P(k/Z)* (of size *Z+1*). The probabilities entering the tridiagonal transition matrix $S=\left[p_{ij}\right]^T$ are defined as $p_{k,k\pm 1}=T_\mu^\pm(k)$ and $p_{k,k}=1-p_{k,k-1}-p_{k,k+1}$, where $T_\mu^\pm$ stands for the transition probabilities for an arbitrary mutation rate μ, which are given by $T_\mu^+(k)=(1-\mu)T^+(k)+\mu(Z-k)/Z$ for the probability to increase from k to *k+1* Cs and $T_\mu^-(k)=(1-\mu)T^-(k)+\mu\, k/Z$ for the probability to decrease to *k-1* [56]. The stationary distribution is then obtained from the eigenvector corresponding to the eigenvalue 1 of S [53, 60].

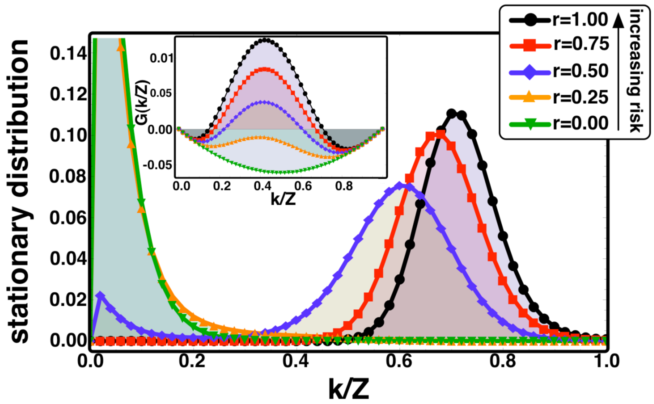

Figure 4. Prevalence of cooperation in finite populations. The main panel pictures the stationary distribution corresponding to the prevalence of each fraction of *C*s that emerges from the discrete gradient of selection *G* shown in inset. Whenever risk is high, stochastic effects turn collective cooperation into a pervasive behavior, rendering cooperation viable and favoring the overcome of coordination barriers, irrespective of the initial configuration (*Z=50*, *N=6*, *M=3*, *c=0.1*, *μ=0.005*).

In Fig. 4 we show the stationary distributions for different values of risk, for a population of size *Z=50* where *N=2M=6*. While the finite population gradient of selection *G(k/Z)* shown in the inset exhibits a behavior qualitatively similar to $\dot{x}$ in Fig. 1, the stationary distributions show that the population spends most of the time in configurations where *C*s prevail, irrespectively of the initial condition. This is a direct consequence of stochastic effects, which allow the “tunneling” through the coordination barrier associated with x_L, rendering such coordination barrier (x_L) irrelevant and turning cooperation into the prevalent strategy. On the other hand, the existence of a stable fixed root of *G* is triggered in *P* with a maximum at this position, unlike what one observes with ϕ_k.

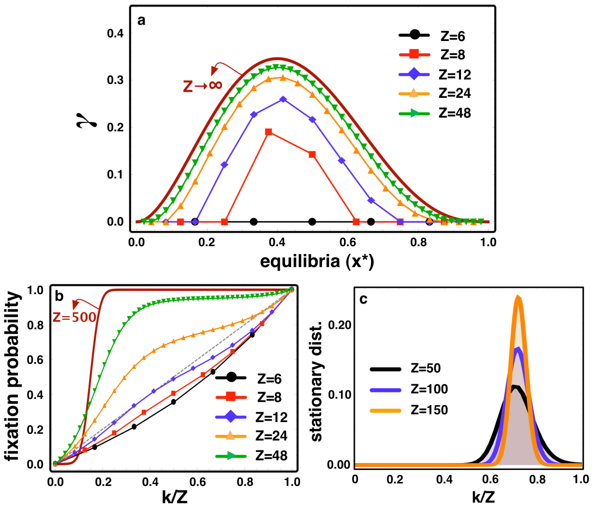

Figure 5. Population size dependence for *N=6=2M*. a) Roots of the gradient of selection for different values of the cost-to-risk ratio and population sizes. b) Fixation probabilities for different values of the population size for a fixed cost-to-risk ratio (γ=0.1) as a function of the number of *C*s (β=5.0). c) We introduce a small mutation (μ=0.005) to show the stationary distribution for the same game parameters in b) and different population sizes. As the population size increases, the system spends increasingly less time close to the monomorphic configurations.

Yet, until now the effect of the population size on the game itself remains uncharted. In Fig. 5a, we plot the roots of *G(k)* as a function of the *cost-to-risk ratio* for different values of population size *Z*. For large *Z* the general picture described for infinite populations remains qualitatively valid. As before, two interior roots of *G(k)* characterize the evolutionary dynamics of the population. However, the position of the interior fixed points can be profoundly altered by the population size. The range of *k/Z* in which *C*s are advantageous is also strongly reduced for small populations. Moreover, while $\bar{x}$ (see section 2) remains almost unchanged as we move from infinite to finite populations, the critical $\bar{\gamma}$ is drastically reduced for small populations that, in turn, reduces the interval of *cost-to-risk ratios* for which a defection dominance dilemma is replaced by a combination of coordination and co-existence dilemmas. In other words, the smaller the population size the higher the perception of risk needed to achieve cooperation. The population size also plays an important role on the shape of the stationary distribution: In Fig. 5c we plot the stationary distribution for $r=1$ and $c=0.1$, for different population sizes. Whenever the population size increases, a higher number of errors is needed to escape the equilibrium between *C*s and *D*s, leading the system to spend a higher fraction of time on the internal stable root of $G(k)$.

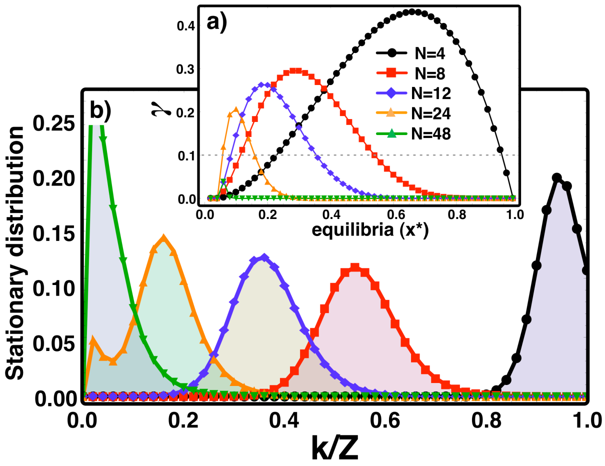

Figure 6. Group size dependence for $M=3$. a) Roots of the gradient of selection for different values of the cost-to-risk ratio and group sizes. b) Stationary distribution for different group sizes and $c/r=0.15$. Cooperation will be maximized in small groups, where the risk is high and goal achievement involves stringent requirements.

Naturally, the assessment of the effects of the population size should be carried out in combination with the number of parties involved in collective-risk dilemmas, i.e., the group size. Whether game participants are world citizens, world regions or country leaders, it remains unclear at which scale global warming should be tackled [40, 61]. Indeed, besides perception of risk, group size may play a pivotal role when maximizing the likelihood of reaching overall cooperation. As shown by the stationary distributions in Fig. 6, cooperation is better dealt with within small groups, with the proviso that for higher M/N

values, coordination is harder to attain, as shown by the position of the roots of G (see inset of Fig. 6).

5. Conclusions

Dealing with environmental sustainability cannot overlook the uncertainty associated with a collective investment. Here we propose a simple form to describe this problem and study its impact in behavioral evolution, obtaining an unambiguous agreement with recent experiments [9], together with several concrete predictions. We do so in the framework of non-cooperative N-person evolutionary game theory, an unusual mathematical tool within the framework of modeling of political decision-making. We propose a new N-person game where the risk of collective failure is explicitly introduced by means of a simple collective dilemma. Moreover, instead of resorting to complex and rational planning or rules, individuals revise their behavior by peer-influence, creating a complex dynamics akin to many evolutionary systems. This framework allowed us to address the impact of risk in several configurations, from large to small groups, from deterministic towards stochastic behavioral dynamics.

Overall, we have shown how the emerging behavioral dynamics depends heavily on the perception of risk. The impact of risk is enhanced in the presence of small behavioral mutations and errors and whenever global coordination is attempted in a majority of small groups under stringent requirements to meet coactive goals. This result calls for a reassessment of policies towards the promotion of public endeavors: Instead of world summits, decentralized agreements between smaller groups (small N), possibly focused on region-specific issues, where risk is high and goal achievement involves tough requirements (large relative M) [62], are prone to significantly raise the probability of success in coordinating to tame the planet's climate. Our model provides a "bottom-up" approach to the problem, in which collective cooperation is easier to achieve in a distributed way, eventually involving regions, cities, *NGO*s and, ultimately, all citizens. Moreover, by promoting regional or sectorial agreements, we are opening the door to the diversity of economic and political structure of all parties, which, as showed before [32, 63] can be beneficial to cooperation.

Naturally, we are aware of the many limitations of a bare model such as ours, in which the complexity of Human interactions has been overlooked. From higher levels of information, to non-binary investments, additional layers of realism can be introduced in the model. Moreover, from a mathematical perspective, several extensions and complex aspects common to human socio-economical systems could be further explored [64-67]. On the other hand, the simplicity of the dilemma introduced here, makes it generally applicable to other problems of collective cooperative action, which will emerge when the risks for the community are high, something that repeatedly happened throughout Human history [68, 69], from ancient group hunting to voluntary adoption of public health measures [70-72]. Similarly, other cooperation mechanisms [10, 13, 15, 18, 22-26], known to encourage collective action, may further enlarge the window of opportunity for cooperation to thrive. The existence of collective risks is pervasive in nature, in particular in many dilemmas faced by Humans. Hence, we believe the impact of these results go well beyond decision-making towards global warming.

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