Tracking the Evolution of Cooperation in Complex Networked Populations

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Abstract. Social networks affect in such a fundamental way the dynamics of the population they support that the global, population-wide behavior that one observes often bears no relation to the agent processes it stems from. Up to now, linking the global networked dynamics to such agent mechanisms has remained elusive. Here we define an observable dynamic and use it to track the self-organization of cooperators when co-evolving with defectors in networked populations interacting via a Prisoner's Dilemma. Computations on homogeneous networks evolve towards the coexistence between cooperator and defector agents, while computations in heterogeneous networks lead to the coordination between them. We show how the global dynamics co-evolves with the motifs of cooperator agents in the population, the overall emergence of cooperation depending sensitively on this co-evolution.

Keywords: Complex Networks, Self-Organization, Cooperation, Evolutionary Game Theory, Evolutionary Dynamics.

1 Introduction

Dynamical processes involving populations of agents constitute paradigmatic examples of complex systems. From epidemic outbreaks to opinion formation, evolutionary and learning behavioral dynamics, the impact of the underlying web of ties in the overall behavior of the population is well known [1, 6, 10, 12, 13, 15, 21, 22, 26, 30, 41, 42]. Furthermore, it is often impossible to avoid such structures when applications require the deployment of agents under physical or other constraints as it is with network routing [18,29], computational intelligence techniques [7, 8, 43] and sensor networks [2].

In this context, Evolutionary Games [35] provides one of the most sophisticated examples of complex system dynamics in which the role of the underlying network topology proves ubiquitous. For instance, when cooperation is modeled as a Prisoner's dilemma game (**PD**), cooperation may emerge (or not) depending on how agents are networked [14, 16, 23–25, 27, 31, 32, 37, 38]. Up to now, multi-agent based models were unable to identify the detailed mechanism by which

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local self-regarding actions lead to a collective cooperative scenario, in particular relating it to the network topology. In the following, we devise a means to establish such a link between individual and collective behaviors, in terms of the underlying network topology. To this end we make use of evolutionary game dynamics, although the method should be easily applicable to other dynamical processes taking place on general complex networks.

2 Results and Discussion

2.1 Evolution of Cooperation in Finite Well-Mixed Populations

Let us consider two agents who can each adopt one of two possible behaviors: *Cooperator* (C) or *Defector* (D). Whenever they interact, four outcomes are possible: Two Cs receive R (reward) each, whereas each receives P (punishment) if both are Ds. Whenever a C interacts with a D, the C gets S (the sucker's payoff) whereas the D gets T (temptation to defect). These outcomes can be summarized through the so-called payoff matrix,

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{1}$$

Whenever $T > R > P > S$ one obtains the **PD** [4, 35]. For simplicity, we formalize the **PD** game in terms of a single parameter B (benefit) by defining $T = B > 1$, $R = 1$, $S = 1 - B$ and $P = 0$.

In the context of Evolutionary Game Theory [35], the payoff of an agent is associated with her/his fitness that is her/his social success. Thus, behaviors that provide higher rewards are imitated more frequently and spread in the population. Here, evolution and strategy update is modelled via a stochastic birth-death process in finite populations of size N, often referred as *pairwise comparison* rule [35, 39]. At each iteration, a randomly selected agent x adopts the strategy of a randomly selected neighbor y with probability given by the Fermi distribution

$$p = [1 + e^{-\beta(f_y - f_x)}]^{-1}, \tag{2}$$

where the fitness values f_x (f_y) stand for the accumulated payoff of x (y) and β controls the intensity of selection measuring the importance of the agent payoffs and stochastic effects in the imitation process [39].

In the limit of well-mixed populations of size N – where agents may interact with any other agent in the population –, Cs are always worse off than Ds, and will be outcompeted under natural selection [35]. Mathematically, this means that the gradient of selection [28, 34, 39]

$$G(j) = T^{+}(j) - T^{-}(j) \tag{3}$$

is negative for all j, where j stands for the number of Cs in the population and

$$T^{\pm}(j) = \frac{N-j}{N}\frac{j}{N}\frac{1}{1 + e^{\pm\beta(f_D - f_C)}} \tag{4}$$

represent the probabilities to increase/decrease the number of Cs in the population [40].

The elegance of this result (despite the doomsday scenario for Cs) is best appreciated when realizing that the population ends up adopting the Nash-equilibrium of a **PD** game interaction between two agents: everybody defects. Consequently, there is no difference in the outcome of the game, from an agent or from a (collective) population wide perspective. This result holds in structureless populations, a feature which is seldom observed in practice, with strong implications in many natural phenomena.

It is noteworthy that the general methodology discussed in the next section is independent from the stochastic update rule adopted in the evolutionary process. Moreover, this stochastic update is more general one could initially foresee, as the ensuing dynamics may be also shown to be equivalent to the replicator equation [17, 40] and to finite action learning automata in the limit of infinite, well-mixed populations [9, 36, 41].

2.2 Gradients of Selection in Structured Populations

A homogeneous network, in which all agents engage in the same number of games (k) with their first neighbors, represents the simplest case of a structured population, where agents occupy the nodes of the network, whose links determine who is neighbor of whom. Unlike well-mixed populations, even in such simple homogeneous scenario where all agents share the same number of neighbors, agents with the same strategy no longer necessarily share the same fitness (here associated with game payoff): fitness becomes context-dependent and so does the gradient of selection, which is now impossible to compute analytically.

To overcome this problem, we define the Average Gradient of Selection (***AGoS***), denoting it by $G^A(j)$ as the average *i*) over all possible transitions taking place in every node of the network throughout evolution, and *ii*) over a large number of networked evolutions. For each agent i we compute the probability of changing behavior at time t,

$$T_i = \frac{1}{k_i} \sum_{m=1}^{\bar{n_i}} [1 + e^{-\beta(f_m - f_i)}]^{-1}, \tag{5}$$

where k_i stands for the degree of node i and $\bar{n_i}$ for the number of neighbors of i having a strategy different from that of i. The ***AGoS*** at a given time t of simulation p, where we have j Cs, is defined as,

$$G_p(j,t) = T_A^+ - T_A^- \tag{6}$$

where, $T_A^{\pm} = \frac{1}{N} \sum_{i=1}^{\substack{AllDs \\ AllCs}} T_i(t)$.

For a given network type, we run $\Omega = 2 \times 10^7$ simulations (using 10^3 randomly generated networks) starting from all possible initial fractions j/N of Cs. Each configuration of the population is associated with the fraction j/N of Cs. Evolutions run for $\Delta = 10^5$ time steps. Hence, the overall, time-independent

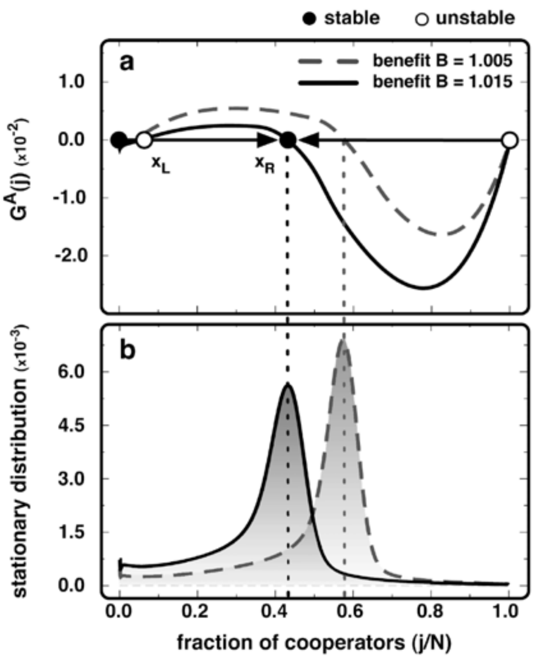

Fig. 1. Time-independent *AGoS*. (a) We plot $G^A(j)$ for a population of players interacting via a **PD** in a homogeneous random network, for two values of the benefit B. Globally, $G^A(j)$ indicates that the population evolves towards a co-existence scenario. **(b)** Stationary distributions showing the pervasiveness of each fraction j/N in time. In line with the ***AGoS*** in a), the population spends most of the time in the vicinity of the stable-like root x_R of $G^A(j)$. When $j/N \approx 0$, Cs become disadvantageous, giving rise to an unstable-like root x_L of $G^A(j)$ which, however, plays a minor role as shown ($N = 10^3$, $k = 4$ and $\beta = 1.0$). Homogeneous random networks were obtained by repeatedly swapping the ends of pairs of randomly chosen links of a regular lattice [33].

AGoS is given by the average

$$G^A(j) = \frac{1}{\Omega\Delta}\sum_{t=1}^{\Delta}\sum_{p=1}^{\Omega} G_p(j,t) \tag{7}$$

over all simulations and time-steps.

The gradient of selection in networks has to be computed numerically and has the nice property of being network dependent but context independent, as it recovers a population most likely direction of selection. As demonstrated below, ***AGoS*** allow us to follow in time the evolutionary dynamics from a global, population-wide perspective, as opposed to an agent perspective, which can always be inferred from the structure of the payoff matrix.

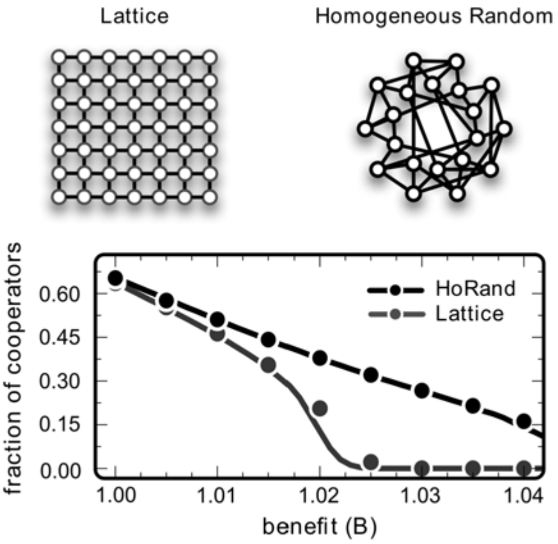

Fig. 2. Evolutionary dynamics cooperation in homogeneous networks. We plot the interior roots x_R of $G^A(j)$ (circles and squares) for a **PD** ($T = B$, $R = 1$, $P = 0$, $S = 1 - B$) in homogeneous networks, from random networks (circles) to ordered lattices (squares), as a function of the benefit B. $G^A(j)$ indicates that the population evolves towards a stationary fraction x_R of Cs. This is confirmed by the stationary states (lines) obtained via computer simulations starting from 50% of Cs and Ds randomly placed in each network. ($N = 10^3$, $k = 4$ and $\beta = 0.1$).

2.3 Results for Homogeneous Networks

The results for $G^A(j)$ on homogeneous random networks are shown in Fig. 1a. Unlike well-mixed populations, where cooperation has no chance and $G^A(j) < 0$ for all values of j, homogeneous networks can sustain cooperation [24, 33, 37]. The shape of $G^A(j)$ suggests that, even though every agent engages in a **PD**, from a global, population-wide perspective, homogeneous networks give rise to an emerging collective dynamics promoting the co-existence between Cs and Ds defined by a co-existence point at $j/N = x_R$.

This hypothesis is confirmed when one computes the stationary distribution, which measures the fraction of time that the population spends in each available state j/N before reaching fixation (Fig. 1b). It represents the pervasiveness in time of each composition of the population [19], here identified by the fraction of Cs. The remarkable agreement between the roots of $G^A(j)$ and the peaks of the stationary distribution gives credit to $G^A(j)$ while emphasizing the fundamental transformation in the evolutionary dynamics of the population introduced by a complex network of interactions. As we show below, the emergence of an unanticipated global (macroscopic) dynamics from a distinct agent (microscopic) dynamics pervades throughout evolutionary dynamical processes in structured populations.

The co-existence point is associated with the internal root of $G^A(j)$, x_R, inexistent in well-mixed populations, and whose location decreases with increasing B. Together with x_R one obtains a coordination root ($x_L \approx 0$) of $G^A(j)$ since, in the absence of cooperative partners, Cs will always be disadvantageous. However, the impact of x_L is minor, as shown in Fig. 1b. In Fig. 2 we track the position of x_R (dots) for two different homogeneous structures along a range of the B values. These are compared with the equilibrium fraction of cooperators (lines), in other words the stationary states. As expected we find a match between the ***AGoS*** prediction and the dynamical outcome, thus providing evidence that the ***AGoS*** remains valid and quantitatively accurate for a broad range of game parameters and different types of homogeneous networks.

Fig. 1a shows that, as we move from a single agent to a population wide perspective, one witnesses the emergence of a new evolutionary dynamics. This new global dynamics has important practical consequences: The fixation time – the time required for Cs to invade the entire population – becomes much larger in homogeneous networks when compared to well-mixed populations (irrespective of the small-world effects associated with random links) as the population spends a large period of time in the vicinity of x_R, mainly when selection is strong (large β).

The analysis in Fig.1 was limited to the time-independent $G^A(j)$ as we averaged over the entire time span of all runs. However, the ***AGoS*** itself evolves in time, giving origin to a time-dependent $G^A(j,t)$. At the beginning of each simulated evolution, Cs and Ds are randomly spread in the network, precluding the occurrence of correlated (assorted) clusters of agentswith the same strategy. Hence, $G^A(j,t=0) < 0$ in general. As populations evolve, Cs (Ds) breed Cs (Ds) in their neighborhood, promoting the assortment of strategies, with implications both on the fitness of each player and on the shape (and sign) of $G^A(j,t)$. The time-dependent gradients $G^A(j,t)$ for a particular generation t_0 (and corresponding roots) are trivially computed by averaging over the configurations occurring during N previous time-steps (1 generation),

$$G^A(j) = \frac{1}{\Omega\Delta} \sum_{t=t_0-N}^{t_0} \sum_{p=1}^{\Omega} G_p(j,t) \tag{8}$$

In Fig. 3a we plot snapshots of $G^A(j,t)$ for three different times, whereas Fig. 3b portrays the time evolution of the internal roots (x_L and x_R) of $G^A(j,t)$, on which we superimposed two evolutionary runs starting with strategies randomly placed in the population. As $G^A(j,t=0) < 0$, the fraction of Cs will start decreasing (Fig. 3a). However, with time, strategy assortment leads to the emergence of a co-existence root of $G^A(j,t)$, towards which the fraction of Cs converges. The ensuing coexistence between Cs and Ds, which matches perfectly the shape of $G^A(j,t)$, stems from the evolving self-organization of Cs and Ds in the network, defining a global dynamics which is impossible to predict from the nature of the local (**PD**) interactions.

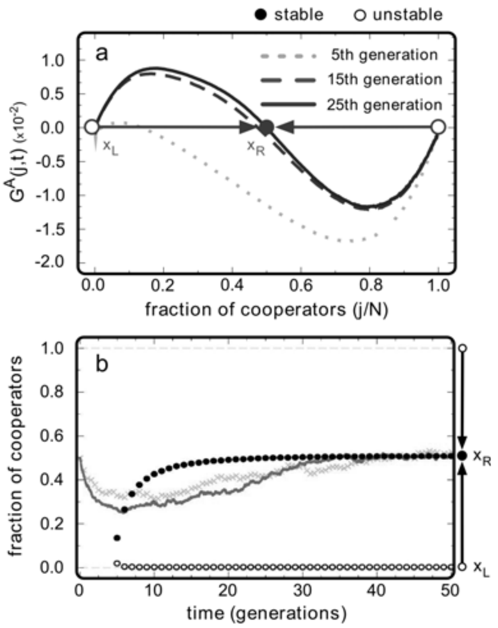

Fig. 3. Time-dependent *AGoS*. (a) We plot $G^A(j,t)$ for three different moments of evolutionary time. Each line provides a snapshot for a given moment, portraying the emergence of a population-wide (time-dependent) co-existence-like dilemma stemming from an agent (time-independent) defection dominant dilemma (**PD**). **(b)** The circles show the position of the different interior roots of $G^A(j,t)$, whereas the solid (dark grey points) line and (light grey crosses) crosses show two independent evolutionary runs starting from 50% of Cs and Ds randomly placed in the networked population. Open (full) circles stand for unstable, x_L (stable, x_R) roots of $G^A(j,t)$ ($B = 1.01$, $N = 10^3$, $k = 4$ and $\beta = 10.0$).

2.4 Results for Heterogeneous Networks

It is now generally accepted that homogeneous networks provide a simplified picture of real interaction networks [3,5,6,11–13]. Most social structures share a marked heterogeneity, where a few nodes exhibit a large number of connections, whereas most nodes comprise just a few. The fingerprint of this heterogeneity is provided by the associated network degree distributions, which exhibit a broad-scale shape, often resembling a power-law [3, 5, 6, 12]. In the following we use $G^A(j,t)$ to show how population heterogeneity shifts the internal roots in Fig. 1 to the right, effectively transforming a co-existence scenario into a coordination one. To this end, we compute $G^A(j,t)$ employing scale-free (**SF**) networks of Barabási and Albert (**BA**) [5], which provide a widely used representation of a heterogeneous structured population [12]. Fig. 4a shows $G^A(j)$ for **BA** networks, whereas the circles in Fig. 4b portray the time evolution of the internal roots of $G^A(j,t)$.

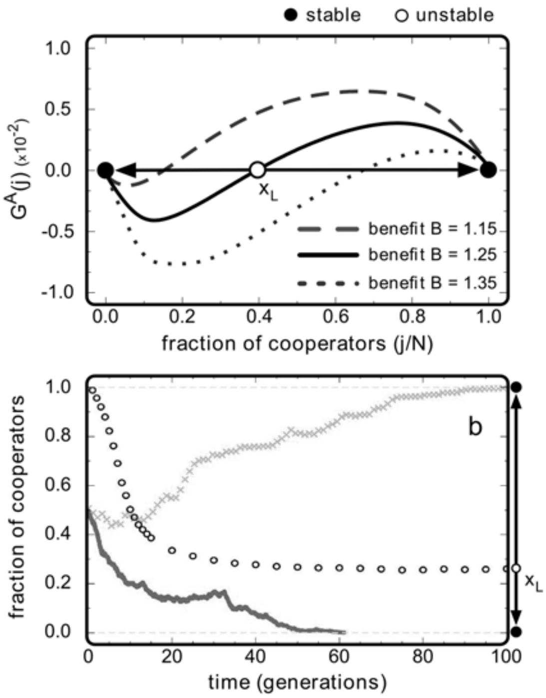

Fig. 4. AGoS on BA networks. (a) Starting from a defection dominance **PD** played at an agent level, a coordination dynamics emerges at a global, population-wide scale, for the three values of B depicted. **(b)** Evolution of the unstable root x_L of $G^A(j,t)$ (open circles), exhibiting the time-dependence of the global dynamics; solid (dark grey dots) line and (light grey crosses) crosses show two independent evolutionary runs starting from 50% of Cs and Ds randomly placed. The ultimate fate of Cs in each run depends on whether the population composition crosses over the time-dependent value x_L of $G^A(j,t)$, thereby overcoming the dynamical coordination barrier during evolution. ($B = 1.25$, $N = 10^3$, $< k >= 4$ and $\beta = 0.1$). BA networks were obtained combining growth and preferential attachment, following the model proposed by Barabási and Albert [5].

Clearly, heterogeneous networks lead to a global dynamics dominated by a coordination threshold x_L. This unstable root of $G^A(j,t)$ represents the critical fraction of Ds above which they are able to assort effectively. Once this happens, they successfully invade highly connected nodes (hubs), rendering cooperation an advantageous strategy, as Cs acquire then a higher probability of being imitated than Ds. The requirement that Cs must first invade hubs before outcompeting Ds (by formation of cooperative star-like clusters [27]), makes invasion harder for isolated Cs. Consequently, the unstable root x_L (located close to $j/N \approx 0$ in homogeneous networks) moves here to higher fractions of Cs. Once this coordination is overcome, Cs benefit from the strong influence of hubs to rapidly spread in the population, eventually leading to fixation. Hence, the stable internal root x_R which characterizes $G^A(j)$ in homogeneous networks collapses into values close to $j = N$ on **SF** networks, leading to full cooperation. Naturally, the location of x_L is an increasing function of B, as shown in Fig. 4a.

The requirement that Cs occupy the hubs to outcompete Ds also leads to an intricate interplay between the time-dependent decline of x_L (see Fig. 4b) and the pervasiveness of Cs in the population. In Fig. 4b we show, with full lines, two evolutions in **BA** networks (for the same conditions): One ends up in full cooperation whereas the other reaches full defection. In the former, the fraction of Cs decreases in time slower than x_L. Hence, a crossover moment is reached, after which $j/N > x_L$. As a result, the population will subsequently reach full cooperation. In the latter, j/N remains always below x_L and the population evolves towards full defection. Clearly, heterogeneous networks lead to the emergence of a global dynamics with time-dependent coordination barriers and basins of attraction, all of which can be characterized using $G^A(j,t)$.

3 Conclusions

Overall, our study shows that behavioral dynamics in social networks can be understood as if the network structure is absent but agents faced a different dilemma: The structural organization of a population of self-regarding agents circumvent the Nash-equilibrium of a cooperation dilemma by creating a new dynamical system globally described by two internal fixed points, x_L (unstable) and x_R (stable). Moreover, such a dynamical system, resulting from agents interacting via a two-person game, cannot be mapped onto a two-person evolutionary game in a well-mixed population. On the contrary, such dynamics resembles that from, e.g., N-person dilemmas [20] in the presence of coordination thresholds [28, 34]. Hence, the global dynamics of a 2-person dilemma in structured populations resembles a time-dependent N-person dilemma, in which the coordination or co-existence features emerge from the population structure itself. In this sense, different network topologies emphasize differently this coexistence/coordination dichotomy. In such a context, the ***AGoS*** proves instrumental in characterizing the emergence of a new population-wide evolutionary dynamics.

In sum it is of our belief that these results, together with the methodology proposed here are of broad interest for areas within the biological and social sciences that extend far beyond the scope of cooperation problems [6, 11–13]. Moreover, we address a core problem common to most complex systems analysis on fields such as biology, social and engineering sciences: describe the link between local and global dynamics in multi-agent systems. From human behaviors, epidemics, collective intelligence or many population-based applications, most can be described as an interaction scheme embedded in a complex network for which a tool such as the ***AGoS*** may help us to anticipate the emergent, population-wide, global dynamics.

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