Evolutionary Dynamics of Cooperation Under the Distributed Prisoner's Dilemma

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Abstract. Humans contribute to a broad range of cooperative endeavors. In many of them, the amount or effort contributed often depends on the social context of each individual. Recent evidence has shown how modern societies are grounded in complex and heterogeneous networks of exchange and cooperation, in which some individuals play radically different roles and/or interact more than others. We show that such social heterogeneity drastically affects the behavioral dynamics and promotes cooperative behavior, whenever the social dilemma perceived by each individual is contingent on her/his social context. The multiplicity of roles and contributions induced by realistic population structures is shown to transform an initial defection dominance dilemma into a coordination challenge or even a cooperator dominance game. While locally defection may seem inescapable, globally there is an emergent new dilemma in which cooperation often prevails, illustrating how collective cooperative action may emerge from myopic individual selfishness.

Key words: Cooperation, Complex Networks, Self-Organization, Evolutionary Game Theory.

1 Introduction

Quite often we are confronted with situations in which the act of giving is more important than the amount given.Take for instance a charity event. Some celebrities are usually invited to participate. Their appearance is given maximal audience, and shown contributing a large amount of money with their media coverage, which is impressive to many, promoters hope to induce a large number of (much smaller) contributions from anonymous (non-celebrities, the overwhelming majority) charity participants, who feel compelled to contribute given the fact that their role model (the celebrity) contributed. Clearly the majority imitates the act of giving and not the amount given.

Many other examples from real life could be provided along similar lines, from trivia, to fads, to stock markets, to Humanitarian causes up to the salvation of planet Earth [1, 2]. Many of these situations provide examples of public goods games (PGG) [3] which are often hard to dissociate from reputation building, social norms and moral principles [4, 5, 6, 7, 8].

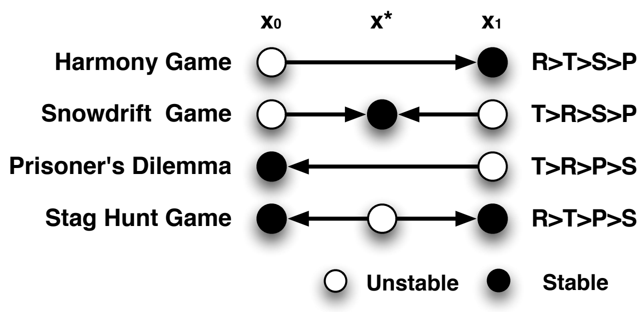

Fig. 1. Replicator Equation fixed points for the four main dilemmas defined by the ranking between the payoff matrix elements: Reward for mutual cooperation (R), Temptation to defect (T), Sucker's Payoff (S) and Punishment for mutual defection (P).

When two individuals meet each other in a one shot game where each one can choose between two possible strategies we are in the presence of a situation well known from game theory as a 2 person dilemma involving two strategies. The possible outcomes of such interaction can be summarized in a 2x2 payoff matrix. If the two strategies are to Cooperate and to Defect we face a dilemma of cooperation and it is common to write the possible outcomes in the payoff matrix as:

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{1}$$

where R (Reward) and P (Punishment) are the payoffs for mutual cooperation and mutual defection respectively, whereas S (Sucker's) and T (Temptation) are the payoffs associated with cooperation and defection when the players use different strategies. Depending on the relative value of these four payoffs, four different dilemmas can be defined as illustrated in Figure 1. These dilemmas cover a wide range of situations one encounters in the real world, from social to microbial interactions. In the Evolutionary Game Theory (EGT) framework, we study the dynamics of populations which are conventionally assumed as infinite and well-mixed. The population dynamics is described by the well known replicator equation, which has two trivial fixed points ($x = 0$, $x = 1$) and may, eventually, have one additional interior fixed point (x^*), see Figure 1.

The simplest PGG involves two individuals. Both have the opportunity to contribute a cost c to a common pool. A Cooperator (C) is one who contributes; otherwise she is a Defector (D).The total amount contributed is multiplied by an enhancement factor F and equally shared among the two participants. Hence, player i (i= 1, 2) using strategy s_i (s_i= 1 if C, 0 if D) gets a payoff $P_i = Fc(s_1 + s_2)/2 - cs_i$ from this game, leading to the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (F-1)/c & Fc/2-c \\ Fc/2 & 0 \end{pmatrix} \end{array} \tag{2}$$

for $F \leq 1$ Ds dominate unconditionally. For $F = 2$ no strategy is favored in well mixed populations (neutral drift); yet, for $F > 2$, it is better to play C despite the fact that, in a mixed pair, a D collects a higher payoff than a C. For $1 < F < 2$ the game falls into the famous symmetric one-shot two-person prisoner's dilemma [9], on which many central results have been obtained over the years, in particular in the context of evolutionary game theory [10, 11, 17, 18, 19, 20, 21, 22, 23, 24, 25, 12, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

In the social dilemma with the payoff matrix given by equation (2), every C pays a fixed cost c per game, providing the same benefit b to the partner. However, if what matters is the act of giving and not the amount given, then there is no reason to assume that everybody contributes the same cost c to each game. Depending on the amount of each individual contribution, the overall result of the evolutionary dynamics may change. The two person game introduced above provides not only the ideal ground to introduce such a diversity of contributions, but also an intuitive coupling between game dynamics and social structure: The first (second) individual contributes a cost c_1 (c_2) if playing C and nothing otherwise. Hence, player i (i = 1, 2) now gets the following payoff from this game:

$$P_i = F(c_1 s_1 + c_2 s_2)/2 - c_i s_i \tag{3}$$

reflecting the symmetry breaking induced by possibly different contributions from different cooperating individuals. This poses a natural question: Who will acquire an evolutionary edge under these conditions?

Often the amount that each individual contributes is correlated with the social context she is actually embedded in [12, 13]. Modern communities are grounded in complex social networks of investment and cooperation, in which some individuals play radically different roles and interact more and more often than others. Empirical studies have demonstrated that social networks share both small-world properties and heterogeneous distribution of connectivities [14, 15, 16]. In such heterogeneous communities, where different individuals may be embedded in very different social environments, it is hard to imagine that every C will always provide the same amount in every game interaction, hence reducing the problem to the standard two-person prisoner's dilemma studied so far. In the context of N-person games played in prototypical social networks, it has been found that the diversity of contributions greatly favors cooperation. However, and similar to the relation between two-body and many-body interactions in the Physical Sciences, N-person public goods games have an intrinsic complexity which cannot be anticipated from two-person games. As such it is not clear in which way heterogeneous networks, which naturally induce the symmetry breaking alluded to before, enhance or inhibit the evolution of cooperation.

2 MODEL AND METHODS

We will consider finite populations structured by means of complex networks, where each node represents an individual, with links representing interactions between them.

Our study will be focused on three topologies: Scale-Free (SF), generated using a direct implementation of the Barabási-Albert (BA) model based on growth and preferential attachment [36]; Exponential (EXP), generated replacing the preferential attachment by uniform attachment in the BA model. Both these networks are heterogeneous, as in general different nodes will exhibit different connectivities. Finally, we shall also consider Regular networks (REG), which are homogeneous in the sense that all nodes exhibit the same connectivity. It is known that social networks fall somewhere between the limits of SF and REG networks [37], and hence the choice of studying an intermediate class represented by the EXP networks.

We will study two regimes in the framework of the Prisoner's Dilemma: the conventional one (CPD) where each C contributes the same cost c to each game she plays, and the distributed one (DPD) where Cs now equally distribute a given cost c among all games they play.

In each time-step, every individual engages in a 2-person PGG with each of his neighbors. The accumulated payoff from all interactions is associated with reproductive fitness (f_i) or social success, which determines the behavior in the next generation. We adopt the so-called pairwise comparison rule [38, 39, 40] for the social learning dynamics: Each individual (x) copies the behavior of a randomly chosen neighbor (y) with a probability given by the Fermi distribution from statistical physics:

$$p = (1 + e^{-\beta(f_y - f_x)})^{-1} \tag{4}$$

in which β , known as the intensity of selection, plays the same role as the inverse of temperature in Physics. By this definition the probability increases with the fitness difference. For $\beta \ll 1$ we fall into the weak selection regime, as selection provides a small perturbation to random drift; opposite to this scenario, imitation dynamics is obtained for high values of β. Throughout this work we adopt the strong selection regime ($\beta = 10.0$) as this enhances both the influence of the payoff values in the individual fitness and the role played by the social network.

In all our simulations we will keep $c = 1$.

3 RESULTS

Figure 2 shows the final fraction of cooperators, corresponding to the average, over 10^3 runs and networks, of the value obtained for this fraction, in each run, after 10^5 generations. This is done for each point (F and degree) and for the three different population structures.

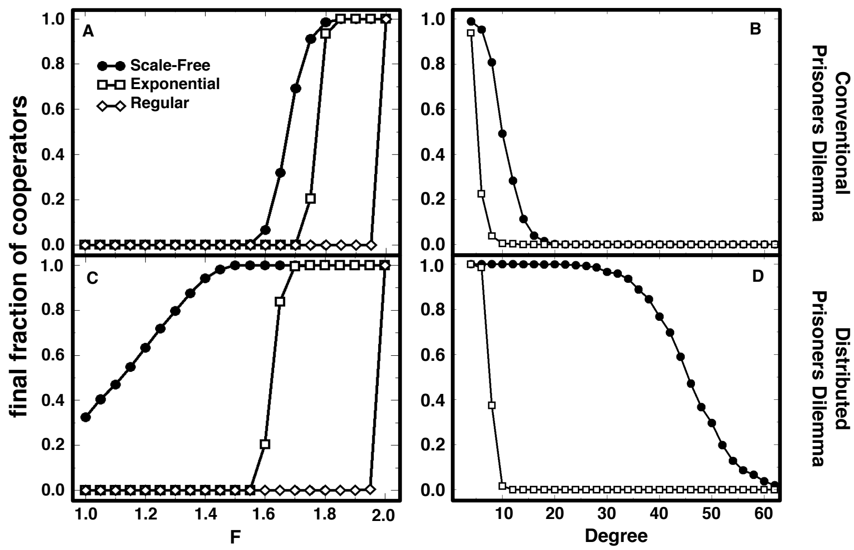

Fig. 2. Final fraction of cooperators as a function of i) the enhancement factor F (panels A and C) and ii) the degree for the EXP and SF networks with F = 1.8 (panels B and D). Panel A: Under CPD Cooperation is able to dominate on SF networks (filled circles), unlike what happens on REG structures (empty squares). On exponential networks, intermediate levels of cooperation emerge, as a result of the heterogeneity of such topologies (empty diamonds). Panel C: Under DPD the advantage of Cs is dramatically enhanced when the same cost is evenly shared among each neighbor. As expected, abandoning the well-mixed regime leads to a break-up of neutrality for $F = 2$. Panels B and D : Cooperation is able to dominate on sparse networks. Yet only under DPD, combined with high levels of heterogeneity attained on Scale-free networks, one observes the maintenance of cooperative behavior in highly connected populations. The results were obtained for networks of 10^3 nodes and variable average degree ($z = 4$ in panels A and C) starting with 50% of Cs randomly distributed in the population.

Figure 2A shows the outcome of evolving the conventional 2-person PD ($1 < F < 2$), in which case each player contributes a fixed amount c to each game she participates, the existence of a minority of highly connected individuals in SF networks (line and filled circles) allows the population to preserve high cooperative standards, while on homogeneous networks (line and empty diamonds), Ds dominate for the entire range of parameters, as a result of the pairwise comparison rule adopted [41]. Heterogeneous networks thus pave the way for the emergence of cooperation. Highly connected individuals (i.e. hubs) work as catalysers of cooperative behavior, as their large number of interactions allows them to accumulate a high fitness. This, in turn, leads them to act as role models for a large number of social ties. To the extent that hubs are Cs, they influence the vast majority of the population to follow their behavior. Clearly, this feature has a stronger impact on SF networks than on EXP networks, the

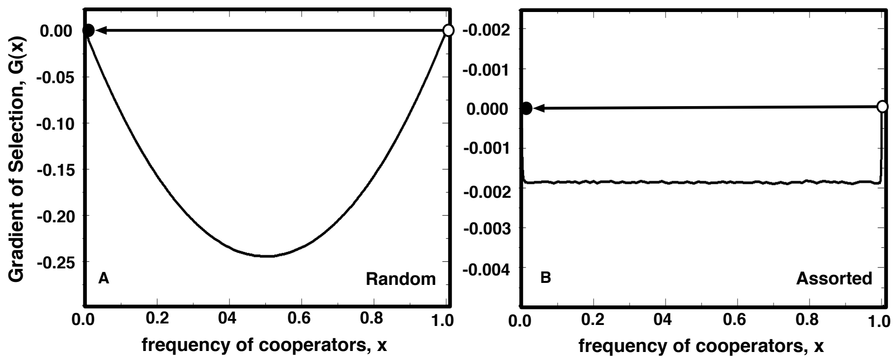

Fig. 3. The Gradients of Selection of Regular Networks for F = 1.25 for both random and assorted distributions of strategies. For both distributions the G(x) is always negative, meaning, that the features of in the initial prisoner's dilemma remain intact at a population-wide level.

difference between these two types of networks stemming from the presence or absence, respectively, of the preferential attachment mechanism.

The results in Figure 2A and 2B are based on a CPD assumption while Figure 2C and 2D considers the DPD dilemma. While on homogeneous networks the fate of cooperation is the same as before - it amounts to rescaling the intensity of selection - heterogeneity in the amount contributed by each individual to each game creates a remarkable boost in the final number of Cs for the entire range of F, which increases with increasing heterogeneity of the underlying network. Comparison with the results of Figure 2A shows that under DPD preferential attachment plays a prominent role, since it constitutes the network wiring mechanism distinguishing EXP networks from SF networks. Changing from CPD to DPD induces moderate boosts in the equilibrium fraction of Cs on EXP networks, but a spectacular boost of cooperation on SF networks: Hubs become extremely influential under the DPD.

The previous analysis shows that heterogeneous networks boost cooperation (Figure 2). A close-up analysis to how the structures that make up such heterogeneous networks, the hubs, interact with each other gives us a glimpse of the way in which they favor cooperation, as a result of the symmetry breaking game dynamics [42]; the same analysis carried out on homogeneous networks is useless as symmetry is kept intact on such networks.

In order to probe deeper into the mechanism(s) underlying the prevalence of cooperators in the DPD, we start by defining the finite population analog $G(x)$ of the gradient of selection under the replicator dynamics $(x(1-x)(f_c - f_d))$: $G(x) = T^{+}(x) - T^{-}(x)$, where $T^{+}(x)(T^{-}(x))$ is the average frequency of transitions increasing (decreasing) the number of Cs for each random configuration with xN Cs, valid for any population size and structure . This is a mean-field variable. Consequently, doing so we overlook the microscopic details of the competition and self-organization of Cs and Ds, but we gain an overview of the game dynamics in a mean-field perspective. G becomes positive whenever cooperation is favored by evolution and negative otherwise. Whenever $G = 0$, selection becomes neutral and evolution proceeds by mainly random drift. Naturally, G will depend implicitly on the population structure, on the fraction x of Cs and also on how these Cs are spread in the network.

At start each individual in the population is assigned a strategy (C or D) randomly, with equal probability, such that correlations between individuals with the same strategy are not favored.When the population evolves such correlations are expected to build up, as *Cs breed Cs* and *Ds breed Ds* [29]. Hence one expects that a possible outcome of evolution is the assortation of strategies where each C(D) has, at least, one C(D) in his neighborhood. This distribution of Cs and Ds replicates the observations of all numerical simulations we have done. For that reason we will always compare the results of random strategy configurations against $G(x)$ of assorted strategy configurations (Figures 3 and 4).

In Figures 3 and 4 we plot the $G(x)$ as a function of x for both strategy distributions, for two different values of F and both prisoner's dilemma paradigms (CPD and DPD).

On homogeneous networks (Figure 3) Ds are always advantageous. On heterogeneous networks the initial prisoner's dilemma (dashed lines) is effectively transformed into a different game (full lines). Figure 4A and 4B indicates that, in the case of CPD, introducing diversity in roles and positions in the social network effectively leads to a coordination game, characterized (in an infinite, well-mixed population) by a critical fraction x^* above which Cs are always advantageous ($G < 0$ for $x < x^*$ and $G > 0$ for $x > x^*$, Figure 1). This result provides a powerful qualitative rationale for many results obtained previously on heterogeneous networks under strong selection [21, 22, 12] in which degree heterogeneity is shown to induce cooperative behavior, inasmuch as the initial fraction of Cs is sufficient to overcome the coordination threshold. Moreover, Figure 4C shows that changing the contributive scheme from CPD to DPD in SF population structures acts to change a prisoner's dilemma effectively into a Harmony game where Cs become advantageous irrespectively of the fraction of Cs ($x^* \approx 0$).

CONCLUSION

The present study puts in evidence the impact of breaking the symmetry of cooperative contributions to the same two-person game. On strongly heterogeneous networks, the results of Figure 2 provide an impressive account of the impact of this diversity of contributions. Our results suggest that whenever the act of cooperation is associated to the act of contributing, and not to the amount contributed, cooperation blooms insofar as the structure of the social web is heterogeneous, leading individuals to play diverse roles. The multiplicity of roles and contributions induced by the social structure effectively transforms a local cooperative dilemma into a global coordination game [43] our results provides

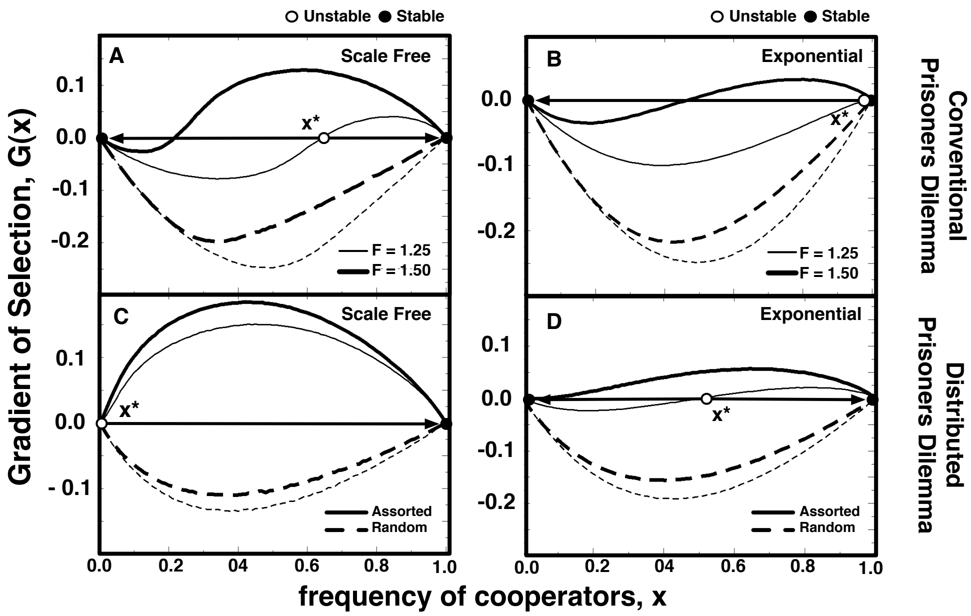

Fig. 4. Gradients of Selection G(x) for $F = 1.25$ (thin lines) and $F = 1.50$ (thick lines) for random (dashed lines) and assorted distribution (solid lines) of strategies for both types of heterogeneous networks (EXP and SF). Under the CPD paradigm (panels A and B) and with the appropriate value of F, heterogeneous networks lead to the appearance of an unstable fixed point $x*$ (open circles) characteristic of a coordination game of the evolutionary dynamics that assorts strategies. Under DPD (panels C and D), the change in the effective game is even more marked and in the case of SF networks the game transformation occurs between a G(x) always negative (prisoner's dilemma) to a scenario where its positive for most values of x leading to a scenario characteristic of a Harmony game, where cooperators dominate unconditionally. In both panels the networks employed had 10^3 nodes and an average degree z = 4, and $\beta = 10.0$.

additional evidence that the assortation of strategies arising from the intricate nature of collective dynamics of cooperation in a complex network leads to a change in the effective game played by the population and for that reason, while locally cooperation can be understood as a prisoner's dilemma, globally it is effectively described by a coordination dilemma[43].

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